SHOCKS

At a shock front propagating in a magnetized fluid at an angle θ with respect to the magnetic induction $\mathbf{B}$, the jump conditions are [13,14]

(1) $\rho U = \bar{\rho}\bar{U} \equiv q$;

(2) $\rho U^2 + p + {B_\perp}^2/2\mu = \bar{\rho}\bar{U}^2 + \bar{p} + {\bar{B}_\perp}^2/2\mu$;

(3) $\rho UV - B_\parallel B_\perp/\mu = \bar{\rho}\bar{U}\bar{V} - \bar{B}_\parallel \bar{B}_\perp/\mu$;

(4) $B_\parallel = \bar{B}_\parallel$;

(5) $UB_\perp - VB_\parallel = \bar{U}\bar{B}_\perp - \bar{V}\bar{B}_\parallel$;

(6) $\frac{1}{2}(U^2 + V^2) + w + (U{B_\perp}^2 - VB_\parallel B_\perp)/\mu\rho U$

$\qquad = \frac{1}{2}(\bar{U}^2 + \bar{V}^2) + \bar{w} + (\bar{U}{\bar{B}_\perp}^2 - \bar{V}\bar{B}_\parallel \bar{B}_\perp)/\mu\bar{\rho}\bar{U}$.

Here U and V are components of the fluid velocity normal and tangential to the front in the shock frame; $\rho = 1/v$ is the mass density; p is the pressure; $B_\perp = B\sin\theta$, $B_\parallel = B\cos\theta$; μ is the magnetic permeability ($\mu = 4\pi$ in cgs units); and the specific enthalpy is $w = e + pv$, where the specific internal energy e satisfies $de = Tds - pdv$ in terms of the temperature T and the specific entropy s. Quantities in the region behind (downstream from) the front are distinguished by a bar. If $\mathbf{B} = 0$, then[15]

(7) $U - \bar{U} = [(\bar{p} - p)(v - \bar{v})]^{1/2}$;

(8) $(\bar{p} - p)(v - \bar{v})^{-1} = q^2$;

(9) $\bar{w} - w = \frac{1}{2}(\bar{p} - p)(v + \bar{v})$;

(10) $\bar{e} - e = \frac{1}{2}(\bar{p} + p)(v - \bar{v})$.

In what follows we assume that the fluid is a perfect gas with adiabatic index $\gamma = 1 + 2/n$, where n is the number of degrees of freedom. Then $p = \rho RT/m$, where R is the universal gas constant and m is the molar weight; the sound speed is given by ${C_s}^2 = (\partial p/\partial\rho)_s = \gamma pv$; and $w = \gamma e = \gamma pv/(\gamma - 1)$. For a general oblique shock in a perfect gas the quantity $X = r^{-1}(U/V_A)^2$ satisfies[14]

(11) $(X - \beta/\alpha)(X - \cos^2\theta)^2 = X\sin^2\theta \left\{[1 + (r-1)/2\alpha]\,X - \cos^2\theta\right\}$, where $r = \bar{\rho}/\rho$, $\alpha = \frac{1}{2}[\gamma + 1 - (\gamma - 1)r]$, and $\beta = {C_s}^2/{V_A}^2 = 4\pi\gamma p/B^2$.

The density ratio is bounded by

(12) $1 < r < (\gamma + 1)/(\gamma - 1)$.

If the shock is normal to $\mathbf{B}$ (i.e., if $\theta = \pi/2$), then

(13) $U^2 = (r/\alpha)\left\{{C_s}^2 + {V_A}^2[1 + (1 - \gamma/2)(r - 1)]\right\}$;

(14) $U/\bar{U} = \bar{B}/B = r$;

(15) $\bar{V} = V$;

(16) $\bar{p} = p + (1 - r^{-1})\rho U^2 + (1 - r^2)B^2/2\mu$.

If $\theta = 0$, there are two possibilities: switch-on shocks, which require $\beta < 1$ and for which

(17) $U^2 = r{V_A}^2$;

(18) $\bar{U} = {V_A}^2/U$;

(19) ${\bar{B}_\perp}^2 = 2{B_\parallel}^2(r - 1)(\alpha - \beta)$;

(20) $\bar{V} = \bar{U}\bar{B}_\perp/B_\parallel$;

(21) $\bar{p} = p + \rho U^2(1 - \alpha + \beta)(1 - r^{-1})$,

and acoustic (hydrodynamic) shocks, for which

(22) $U^2 = (r/\alpha){C_s}^2$;

(23) $\bar{U} = U/r$;

(24) $\bar{V} = \bar{B}_\perp = 0$;

(25) $\bar{p} = p + \rho U^2(1 - r^{-1})$.

For acoustic shocks the specific volume and pressure are related by

(26) $\bar{\upsilon}/\upsilon = [(\gamma + 1)p + (\gamma - 1)\bar{p}] \,/\, [(\gamma - 1)p + (\gamma + 1)\bar{p}]$.

In terms of the upstream Mach number $M = U/C_s$,

(27) $\bar{\rho}/\rho = \upsilon/\bar{\upsilon} = U/\bar{U} = (\gamma + 1)M^2/[(\gamma - 1)M^2 + 2]$;

(28) $\bar{p}/p = (2\gamma M^2 - \gamma + 1)/(\gamma + 1)$;

(29) $\bar{T}/T = [(\gamma - 1)M^2 + 2](2\gamma M^2 - \gamma + 1)/(\gamma + 1)^2 M^2$;

(30) $\bar{M}^2 = [(\gamma - 1)M^2 + 2]/[2\gamma M^2 - \gamma + 1]$.

The entropy change across the shock is

(31) $\Delta s \equiv \bar{s} - s = c_\upsilon \ln[(\bar{p}/p)(\rho/\bar{\rho})^\gamma]$,

where $c_\upsilon = R/(\gamma - 1)m$ is the specific heat at constant volume; here R is the gas constant. In the weak-shock limit ($M \to 1$),

(32) $$\Delta s \to c_\upsilon \frac{2\gamma(\gamma - 1)}{3(\gamma + 1)}(M^2 - 1)^3 \approx \frac{16\gamma R}{3(\gamma + 1)m}(M - 1)^3.$$

The radius at time t of a strong spherical blast wave resulting from the explosive release of energy E in a medium with uniform density ρ is

(33) $R_S = C_0(Et^2/\rho)^{1/5}$,

where C_0 is a constant depending on γ. For $\gamma = 7/5$, $C_0 = 1.033$.